

ON THE EXACT QUARTIC EFFECTIVE ACTION FOR THE TYPE IIB SUPERSTRING*

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Abstract

We propose a four-point effective action for the graviton, antisymmetric two-forms, dilaton and axion of type IIB superstring in ten dimensions. It is explicitly $SL(2, \mathbf{Z})$ -invariant and reproduces the known tree-level results. Perturbatively, it has only one-loop corrections for the NS-NS sector, generalizing the non-renormalization theorem of the R^4 term. Finally, the non-perturbative corrections are of the expected form, namely, they can be interpreted as arising from single D-instantons of multiple charge.

*Research supported in part by the EEC under TMR contract ERBFMRX-CT96-0090.

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1. Introduction

There is a lot of activity nowadays towards understanding the non-perturbative structure in string theory. In type IIB theory, in particular, the non-perturbative physics is intimately related with the existence of the $SL(2, \mathbf{Z})$ symmetry [1, 2]. The spectrum of the type IIB theory contains an $SL(2, \mathbf{Z})$ multiplet of strings and five-branes, the self-dual three-brane, the seven-brane, as well as D-instanton solutions. The latter are the only ones that give non-perturbative corrections in ten dimensions. This can be seen by compactifying the theory. In this case, the various Euclidean $(p + 1)$ -world-volumes of p-branes have an infinite action in the decompactification limit except when $p = -1$, which is just the type IIB D-instanton.

In σ -model perturbation theory, there exists a four-loop divergence that contributes to the β -functions [3] and gives α'^3 corrections to the effective action. This can also be confirmed by string four-point amplitude calculations [4, 6]. For four gravitons, in particular, there exists also a one-loop result [7] for the R^4 corrections and non-renormalization theorems have been conjectured for their structure [8, 9]. One expects that all contributions higher than one loop to vanish, since for higher genus surfaces there are more than eight fermionic zero modes; this is exactly the number needed to saturate the external particles in a four-point amplitude [9]. This heuristic argument has been proved by using superspace techniques [10].

Besides the perturbative corrections to the R^4 term, there also exist non-perturbative ones. Their form has recently been conjectured by Green and Gutperle on the basis of $SL(2, \mathbf{Z})$ invariance [9]. In particular, the modular invariance of the effective action is achieved by employing a certain non-analytic modular form. The structure of the latter is such that it gives only tree- and one-loop corrections to the R^4 term besides the instanton ones. An ansatz for the form of the corresponding four-graviton amplitude has been given in [11]. Moreover, the R^4 term gives rise to a similar term in M-theory [12, 13, 14] and

the compactification of the latter gives results consistent with string theory expectations [15, 16].

One may now proceed further by including the other massless modes of the type IIB theory. In this case, the tree-level result for the four-point amplitudes of the dilaton and the antisymmetric tensor has been given in [6], while a one-loop calculation is lacking. Now, arguments similar to those above seem to suggest that the non-renormalization theorem for the R^4 term may also be extended to the full effective theory when all modes are included. Namely, the perturbative expansion for the NS-NS sector stops at one loop and all other corrections are non-perturbative. This can also be justified by consistency conditions related to M-theory [17]. However, the inclusion of the other modes at the tree level has a serious drawback. It breaks the manifest $SL(2, \mathbf{Z})$ invariance of the theory. Here, we propose an effective action for all bosonic massless modes of type IIB, except for the self-dual four-form. We do not consider the latter because of the lack of any perturbative information at the eight-derivative level. The action we propose respects the $SL(2, \mathbf{Z})$ symmetry and reproduces the effective action of [6] when all R-R fields are switched off. In particular, the NS-NS sector has only tree- and one-loop corrections besides the non-perturbative ones.

In the following section, we recall perturbative results in the type IIB effective theory. In section 3, we summarize the analysis of [9] concerning the non-perturbative corrections to the R^4 term. In section 4, we propose an $SL(2, \mathbf{Z})$ -invariant effective action and discuss its compatibility with a recent calculation [21] of the $R^2(\partial\partial\phi)^2$ term in type IIB on $K3$.

2. Perturbative Effective Type IIB Theory

The massless bosonic spectrum of type IIB superstring theory consists in the graviton g_{MN} , the dilaton ϕ and the antisymmetric tensor B_{MN}^1 in the NS-NS sector and the axion χ , the two-form B_{MN}^2 and the self-dual four-form field A_{MNPQ} in the R-R sector. The two

scalars of the theory can be combined into a complex one, $\tau = \tau_1 + i\tau_2$, defined by

$$\tau = \chi + ie^{-\phi}. \quad (2.1)$$

The theory has two supersymmetries generated by two supercharges of the same chirality. It has in addition a conserved $U(1)$ charge which generates rotations of the two supersymmetries and under which some of the fields are charged [1]. The graviton and the four-form field are neutral, the antisymmetric tensors have charge $q = 1$, whereas the complex scalar τ has $q = 2$. The fermionic superpartners of the above fields are a complex Weyl gravitino and a complex Weyl dilatino.

The bosonic effective Lagrangian of the theory in lowest order in α' takes the form¹

$$\mathcal{L}_0 = R - \frac{1}{2\tau_2^2} \partial_M \tau \partial^M \bar{\tau} - \frac{1}{12\tau_2} (\tau H^1 + H^2)_{KMN} (\bar{\tau} H^1 + H^2)^{KMN}, \quad (2.2)$$

where $H_{KMN}^\alpha = \partial_K B_{MN}^\alpha + \text{cyclic for } \alpha = 1, 2$ and we have set the four-form to zero. The theory has an $SL(2, \mathbf{R})$ symmetry that acts as

$$\tau \rightarrow \frac{a\tau + b}{c\tau + d}, \quad B_{MN}^\alpha \rightarrow (\Lambda^T)^{-1\alpha}{}_\beta B_{MN}^\beta, \quad \Lambda = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbf{R}), \quad (2.3)$$

and leaves the Lagrangian (2.2) invariant. The complex scalar τ parametrizes an $SL(2, \mathbf{R})/U(1)$ coset space. In general, the group $SL(2, \mathbf{R})$ can be represented by a matrix V_\pm^α [1, 9]

$$V = \begin{pmatrix} V_-^1 & V_+^1 \\ V_-^2 & V_+^2 \end{pmatrix} = \frac{1}{\sqrt{-2i\tau_2}} \begin{pmatrix} \bar{\tau} e^{-i\theta} & \tau e^{i\theta} \\ e^{-i\theta} & e^{i\theta} \end{pmatrix}. \quad (2.4)$$

The local $U(1)$ is realized by the shift $\theta \rightarrow \theta + \Delta\theta$ and the global $SL(2, \mathbf{R})$ acts from the left. One may define the quantities

$$P_M = -\epsilon_{\alpha\beta} V_+^\alpha \partial_M V_+^\beta = ie^{2i\theta} \frac{\partial_M \tau}{2\tau_2}, \quad Q_M = -i\epsilon_{\alpha\beta} V_+^\alpha \partial_M V_-^\beta = \partial_M \theta - \frac{\partial_M \tau_1}{2\tau_2}, \quad (2.5)$$

where Q_M is a composite $U(1)$ gauge connection and P_M has charge $q = 2$. We also define the complex three-form

$$G_{KMN} = -\sqrt{2i} \delta_{\alpha\beta} V_+^\alpha H_{KMN}^\beta = -i \frac{e^{i\theta}}{\sqrt{\tau_2}} (\tau H_{KMN}^1 + H_{KMN}^2), \quad (2.6)$$

¹We set $\alpha' = 1$ from now on.

with charge $q = 1$. We fix the gauge by choosing $\theta \equiv 0$ from now on. In this case, the global $SL(2, \mathbf{R})$ transformation is non-linearly realized and the various quantities in eqs.(2.5) and (2.6) transform as

$$P_M \rightarrow \frac{c\bar{\tau} + d}{c\tau + d} P_M, \quad Q_M \rightarrow Q_M + \frac{1}{2i} \partial_M \ln \left(\frac{c\bar{\tau} + d}{c\tau + d} \right), \quad G_{KMN} \rightarrow \left(\frac{c\bar{\tau} + d}{c\tau + d} \right)^{1/2} G_{KMN}. \quad (2.7)$$

We may also define the covariant derivative $D_M = \nabla_M - iqQ_M$, which transforms under $SL(2, \mathbf{R})$ as

$$D_M \rightarrow \left(\frac{c\bar{\tau} + d}{c\tau + d} \right)^{q/2} D_M. \quad (2.8)$$

There exists α'^3 corrections to the effective Lagrangian (2.2) above, which have been evaluated in [6] and are written as

$$\begin{aligned} \mathcal{L}_{4pt} = & \frac{\zeta(3)}{3 \cdot 2^6} \tau_2^{3/2} \left(t_8^{ABCDEFGH} t_8^{MNPQRSTU} + \frac{1}{8} \varepsilon_{10}^{ABCDEFGHIJ} \varepsilon_{10}^{MNPQRSTU}{}_{IJ} \right) \\ & \times \hat{R}_{ABMN} \hat{R}_{CDPQ} \hat{R}_{EFRS} \hat{R}_{GHTU}, \end{aligned} \quad (2.9)$$

where

$$\hat{R}_{MN}{}^{PQ} = R_{MN}{}^{PQ} + \frac{1}{2} e^{-\phi/2} \nabla_{[M} H_{N]}^1{}^{PQ} - \frac{1}{4} g_{[M}^{[P} \nabla_{N]} \nabla^{Q]} \phi. \quad (2.10)$$

The tensor t_8 is defined in [4], ε_{10} is the totally antisymmetric symbol in ten dimensions and the square brackets are defined without the combinatorial factor 1/2 in front. The Lagrangian (2.9) reproduces the four-point amplitude calculated in string theory and it is in agreement with σ -model perturbative calculations. In addition, at the same order in α' one expects contributions coming from five- to eight-point amplitudes. These contributions can be implemented by adding the terms

$$\begin{aligned} & -\frac{1}{4} e^{-\phi} H_{[M}^1{}^{C[P} H_{N]C}^{Q]} + \frac{1}{16} g_{[M}^{[P} \partial_{N]} \phi \partial^{Q]} \phi - \frac{1}{8} g_{[M}^{[P} g_{N]}^{Q]} \partial_K \phi \partial^K \phi \\ & -\frac{1}{4} e^{-\phi/2} \partial^{[P} \phi H^{1Q]}{}_{MN} - \frac{1}{4} e^{-\phi/2} \partial_{[M} \phi H_{N]}^1{}^{PQ} - \frac{1}{8} e^{-\phi/2} g_{[M}^{[P} H_{N]}^1{}^{Q]C} \partial_C \phi \end{aligned} \quad (2.11)$$

to the r.h.s. of eq.(2.10) and then \hat{R}_{PQMN} turns out to be the Riemann tensor associated

to the generalized connection, which includes the torsion and the Weyl connection [6]

$$T_{MNP} = \frac{1}{2}e^{-\phi/2}H_{MNP}^1, \quad \tilde{\omega}_{MNP} = \frac{1}{4}g_{P[N}\partial_{M]}\phi. \quad (2.12)$$

Expanding the \hat{R} terms in eq.(2.9) we find

$$\begin{aligned} \mathcal{L}_{4pt} = & \frac{\zeta(3)}{3 \cdot 2^6} \tau_2^{3/2} \left(t_8 t_8 + \frac{1}{8} \varepsilon_{10} \varepsilon_{10} \right) \left(R^4 + e^{-2\phi} (\nabla H^1)^4 + (\partial\partial\phi)^4 \right. \\ & - 12e^{-\phi} R (\nabla H^1)^2 \partial\partial\phi - 4R^3 \partial\partial\phi + 6e^{-\phi} R^2 (\nabla H^1)^2 \\ & \left. + 6e^{-\phi} (\nabla H^1)^2 (\partial\partial\phi)^2 + 6R^2 (\partial\partial\phi)^2 - 4R (\partial\partial\phi)^3 \right), \end{aligned} \quad (2.13)$$

where $\partial\partial\phi$ stands for $(\partial\partial\phi)_{MNPQ} \equiv g_{MP} \nabla_N \partial_Q \phi$ and ∇H^1 for $(\nabla H^1)_{MNPQ} \equiv \nabla_M H_{NPQ}^1$. It should be noted that there are no terms with odd powers of ∇H^1 , because such terms vanish, owing to the Bianchi identity. One can check for example that the amplitude for three gravitons and one antisymmetric field

$$t_8^{ABCDEFGH} t_8^{MNPQRSTU} R_{ABMN} R_{CDPQ} R_{EFRS} \nabla_{[G} H_{H]TU}^1 \quad (2.14)$$

is zero. This can also be argued on the basis of the invariance of the type IIB superstring under the world-sheet parity that acts on B^1 as $B^1 \rightarrow -B^1$.

At the perturbative level, there exist string one-loop corrections to the four-point functions. For four gravitons these corrections have been calculated [7] and amount to the exchange

$$\zeta(3) \rightarrow \zeta(3) + \frac{\pi^2}{3} \tau_2^{-2}, \quad (2.15)$$

in eq.(2.9).

3. Non-perturbative R^4 couplings

In the type IIB theory, there exist D-instantons that contribute to the four-point amplitudes. Instantons are solutions of the tree-level Lagrangian (2.2) in Euclidean space with vanishing antisymmetric fields and non-trivial profile for the complex scalar τ [18]. In general, they break half of the supersymmetries and the broken ones generate fermionic zero modes. Since the supersymmetry in type IIB theory is generated by a complex Weyl spinor with 16 components, we expect eight fermionic zero modes, which can give a non-zero contribution to four-point amplitudes. In such a background, the action is finite and its value is $S^{(Q)} = -2\pi|Q|i\tau_0$, where Q is the instanton charge and τ_0 is the value of τ at infinity. Thus, we expect the contribution from a single instanton of charge 1 to be proportional to $e^{2\pi i\tau_0}$. The multi-instanton contributions may be determined by T-duality arguments as follows [9]. Under compactification on S^1 , the type IIB D-instanton is mapped to the type IIA D-particle. There are arguments to support the fact that n such single charged D-particles combine to a single bound state of charge n [19]. The world line of this bound state can wrap m times around the compact S^1 so that its topological charge is mn . Then, its T-dual counterpart in type IIB should be a D-instanton of charge $Q = mn$ whose contribution is proportional to $e^{2i\pi|mn|\tau_0}$. Notice that separated instantons are accompanied by additional fermionic zero modes which can only be soaked up with higher than four-derivative interactions.

For four gravitons the full instanton corrections have been conjectured to take the form [9]

$$\mathcal{L}_{R^4} = \frac{1}{3 \cdot 2\bar{\tau}} f_0(\tau, \bar{\tau}) \left(t_8 t_8 + \frac{1}{8} \varepsilon_{10} \varepsilon_{10} \right) R^4, \quad (3.1)$$

where $f_0(\tau, \bar{\tau})$ is the non-holomorphic modular form [20]

$$f_0(\tau, \bar{\tau}) = \sum'_{m,n} \frac{\tau_2^{3/2}}{|m + n\tau|^3}, \quad (3.2)$$

and the sum extends over integers $(m, n) \neq (0, 0)$. In addition, $f_0(\tau, \bar{\tau})$ has the small τ_2^{-1}

expansion

$$\begin{aligned}
f_0(\tau, \bar{\tau}) &= 2\zeta(3)\tau_2^{3/2} + \frac{2\pi}{3}\tau_2^{-1/2} + 8\pi\tau_2^{1/2} \sum_{m \neq 0, n \geq 1} \left| \frac{m}{n} \right| e^{2i\pi mn\tau_1} K_1(2\pi|mn|\tau_2) \\
&= 2\zeta(3)\tau_2^{3/2} + \frac{2\pi}{3}\tau_2^{-1/2} \\
&\quad + 4\pi^{3/2} \sum_{m, n \geq 1} \left(\frac{m}{n^3} \right)^{1/2} (e^{2i\pi mn\tau} + e^{-2i\pi mn\bar{\tau}}) \left(1 + \sum_{k=1}^{\infty} (4\pi mn\tau_2)^{-k} \frac{\Gamma(k-1/2)}{\Gamma(-k-1/2)k!} \right),
\end{aligned} \tag{3.3}$$

where K_1 is a Bessel function. This proposal for the exact R^4 corrections satisfies several consistency requirements:

- i) $SL(2, \mathbf{Z})$ invariance. The function $f_0(\tau, \bar{\tau})$ is modular-invariant, so that \mathcal{L}_{R^4} is invariant as well.
- ii) It reproduces the correct perturbative expansion.
- iii) The non-perturbative corrections are of the expected form: there are only multiply-charged single D-instanton contributions.

It should be noted that although the proposed form of the R^4 corrections to the effective action satisfy the above constraints, there is no proof for its validity. However, from the type IIA side [13, 12], as well as from lower dimensional compactifications of type IIB [22], there exist strong arguments supporting this form of the R^4 terms. Nevertheless, it does not provide the full four-point Lagrangian since the analogous corrections to the other modes (antisymmetric fields and scalars) are lacking. One should expect that the substitution of the Riemann tensor R_{MNPQ} with the modified one in eq.(2.10), as suggested by the tree-level result, is the full answer. However, in this case, the complex scalar and the antisymmetric fields are included in an non-modular-invariant way, which explicitly breaks the conjectured $SL(2, \mathbf{Z})$ symmetry of the type IIB theory. We will construct below a full $SL(2, \mathbf{Z})$ -invariant effective action.

4. The $SL(2, \mathbf{Z})$ -invariant type IIB effective action

The proposed $SL(2, \mathbf{Z})$ -invariant four-point effective action compatible with the tree-level NS-NS sector, which includes the complex scalar and the antisymmetric two-form fields, is

$$\begin{aligned}
\mathcal{S} = & \frac{1}{2} \int d^{10}x \sqrt{-g} \left\{ R - \frac{1}{2\tau_2^2} \partial_M \tau \partial^M \bar{\tau} - \frac{1}{6} G_{KMN} \bar{G}^{KMN} + \frac{1}{3 \cdot 2^7} (t_8 t_8 + \frac{1}{8} \varepsilon_{10} \varepsilon_{10}) \times \right. \\
& \left[\frac{1}{2} f_0(\tau, \bar{\tau}) \left(R^4 + 12 R^2 DP D\bar{P} - 6 R DP DG \bar{G}^2 + 3 R^2 DG D\bar{G} \right. \right. \\
& \left. \left. + 6 DP^2 D\bar{P}^2 + \frac{3}{8} DG^2 D\bar{G}^2 + 6 DP D\bar{P} DG D\bar{G} \right) \right. \\
& \left. + f_1(\tau, \bar{\tau}) \left(-4 R^3 DP + \frac{3}{2} R^2 DG^2 - 12 R DP^2 D\bar{P} - 6 R DP DG D\bar{G} \right. \right. \\
& \left. \left. + 3 DP D\bar{P} DG^2 + \frac{3}{2} DP^2 D\bar{G}^2 + \frac{1}{4} DG^3 D\bar{G} \right) \right. \\
& \left. + f_2(\tau, \bar{\tau}) \left(6 R^2 DP^2 - 3 R DP DG^2 + 4 DP^3 D\bar{P} + 3 DP^2 DG D\bar{G} + \frac{1}{16} DG^4 \right) \right. \\
& \left. \left. + f_3(\tau, \bar{\tau}) \left(-4 R DP^3 + \frac{3}{2} DP^2 DG^2 \right) + f_4(\tau, \bar{\tau}) DP^4 + c.c. \right] \right\}, \tag{4.1}
\end{aligned}$$

where DG stands for $(DG)_{MNPQ} \equiv D_M G_{NPQ}$, DP for $(DP)_{MNPQ} \equiv g_{MP} D_N P_Q$ and similarly for $D\bar{G}$ and $D\bar{P}$ of $U(1)$ charge $q = -1, -2$, respectively. The functions $f_k(\tau, \bar{\tau})$ are defined as

$$f_k(\tau, \bar{\tau}) = \sum'_{m,n} \frac{\tau_2^{3/2}}{(m + n\tau)^{3/2+k} (m + n\bar{\tau})^{3/2-k}}, \tag{4.2}$$

for $k \in \mathbf{Z}$. They transform under $SL(2, \mathbf{Z})$ as

$$f_k(\tau, \bar{\tau}) \rightarrow \left(\frac{c\tau + d}{c\bar{\tau} + d} \right)^k f_k(\tau, \bar{\tau}), \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbf{Z}). \tag{4.3}$$

These functions have the correct modular transformation properties to render the action $SL(2, \mathbf{Z})$ -invariant. Moreover, the effective action we propose satisfies all the criteria listed in the previous section. Namely, it has only tree-level and one-loop perturbative corrections

in the NS-NS sector, which is an extension of the non-renormalization theorem of the R^4 term. This can be seen by examining the small τ_2^{-1} expansion of f_k , which follows from

$$\left(k + 2i\tau_2 \frac{\partial}{\partial \tau}\right) f_k = \left(\frac{3}{2} + k\right) f_{k+1}. \quad (4.4)$$

As a result we obtain

$$f_k(\tau, \bar{\tau}) = 2\zeta(3)\tau_2^{3/2} + c_k\tau_2^{-1/2} + \dots, \quad (4.5)$$

where

$$c_0 = \frac{2\pi^2}{3}, \quad \text{and} \quad c_{k+1} = \frac{2k-1}{2k+3}c_k, \quad k = 0, 1, \dots, \quad (4.6)$$

and the dots in eq.(4.5) stand for instanton corrections.

The action (4.1) we propose has been constructed from the tree-level one (2.13) by replacing

$$\nabla_M \partial_N \phi \rightarrow D_M P_N + D_M \bar{P}_N \quad (4.7)$$

for the dilaton and

$$e^{-\phi/2} \nabla_M H_{NPQ}^1 \rightarrow -\frac{i}{\sqrt{2}} D_M G_{NPQ} + \frac{i}{\sqrt{2}} D_M \bar{G}_{NPQ} \quad (4.8)$$

for the NS-NS antisymmetric tensor. Notice that when $\chi = 0, H^2 = 0$, the r.h.s. of eq.(4.8) gives $e^{-\phi/2} \nabla_M H_{NPQ}^1 + e^{-\phi/2} \partial_M \phi H_{NPQ}^1$, where the second term contributes to at least six-point amplitudes, which is not relevant to our discussion. Under modular transformations, each term in the resulting expression is multiplied by a factor $\left(\frac{c\tau+d}{c\bar{\tau}+d}\right)^{q/2}$, which we finally compensate by replacing $2\zeta(3)\tau_2^{3/2}$ by $f_{q/2}(\tau, \bar{\tau})$. Notice that the terms linear or cubic in ∇H^1 , which vanish anyway at tree level in the expansion (2.13) because of the Bianchi identity would have implied additional contributions in (4.1) involving $f_{k/2}$'s with k odd. However, it is easily seen from their definition in eq.(4.2) that these functions vanish identically.

As a non-trivial check of our proposal, there exists a non-perturbative calculation of type IIB on $K3$ for the four-point amplitude involving two gravitons and two dilatons [21].

This term corresponds to

$$A_{hh\phi\phi} \propto R^{ijkl} R_{ijkl} (\partial_\mu \partial_\nu \phi) (\partial^\mu \partial^\nu \phi), \quad (4.9)$$

where the Latin and Greek indices refer to the internal $K3$ and to the six-dimensional spacetime, respectively. By taking the large $K3$ volume limit, one finds that this amplitude is multiplied by the function $f_0(\tau, \bar{\tau})$ in ten dimensions. However, in our case from the action (4.1) this amplitude is proportional to

$$\left(t_8^{ijkl\mu\nu\rho\sigma} t_8^{mnpq\kappa\lambda\varphi\omega} + \frac{1}{8} \varepsilon_{10}^{ijkl\mu\nu\rho\sigma\alpha\beta} \varepsilon_{10}^{mnpq\kappa\lambda\varphi\omega}{}_{\alpha\beta} \right) R_{ijmn} R_{klpq} g_{\nu\lambda} g_{\sigma\omega} \nabla_\mu \partial_\kappa \phi \nabla_\rho \partial_\varphi \phi, \quad (4.10)$$

and can be seen to vanish by a straightforward calculation. This apparent paradox can be avoided² by recalling that there exists an additional contribution to the six-dimensional amplitude. This arises from the the second and third terms in eq.(2.11) in the definition of \hat{R} in eq.(2.10). In that case one obtains the additional contribution

$$\left(t_8^{ijkl\mu\nu\rho\sigma} t_8^{mnpq\kappa\lambda\varphi\omega} + \frac{1}{8} \varepsilon_{10}^{ijkl\mu\nu\rho\sigma\alpha\beta} \varepsilon_{10}^{mnpq\kappa\lambda\varphi\omega}{}_{\alpha\beta} \right) R_{ijmn} R_{klpq} R_{\mu\nu\kappa\lambda} \times \\ \left(\frac{1}{2} g_{\sigma\omega} \partial_\rho \phi \partial_\varphi \phi - g_{\rho\varphi} g_{\sigma\omega} \partial_\gamma \phi \partial^\gamma \phi \right). \quad (4.11)$$

By expanding around the background $g_{MN} = (\eta_{\mu\nu}, g_{mn}^{K3})$ and by partially integrating the two derivatives in $R_{\mu\nu\kappa\lambda}$, we get a term proportional to the r.h.s. of eq.(4.9). Finally, this term should be promoted in the exact $SL(2, \mathbf{Z})$ -invariant Lagrangian to

$$f_0(\tau, \bar{\tau}) R^{ijkl} R_{ijkl} D_\mu P_\nu D^\mu \bar{P}^\nu, \quad (4.12)$$

which indeed reproduces the result of [21].

5. Conclusions

We have conjectured here an S-duality invariant effective four-point action of type IIB theory. The guiding principles we used were basically the $SL(2, \mathbf{Z})$ invariance and the

²We would like to thank I. Antoniadis and B. Pioline for their contribution on this point.

expectation that the perturbative corrections stop at one loop for the NS-NS sector. We found that these principles can be satisfied by using the functions $f_k(\tau, \bar{\tau})$ we introduced. They give the correct perturbative corrections and the non-perturbative ones are of the expected form. Moreover, the vanishing of four-point amplitudes that involve one or three antisymmetric fields is consistent, as we have discussed, with the vanishing of the f_k forms for half-integer k . Finally, the result of [21] is consistent with our proposal since, for the $K3$ compactification they consider, it arises from a five-point term in ten dimensions. Finally, a simple test of our proposal would be an explicit determination of the one-loop coefficients c_k defined in eq.(4.5).

Acknowledgement

We would like to thank I. Antoniadis, E. Kiritsis, C. Kounnas, N. Obers and B. Pioline for very useful discussions. H.P. would like to thank the kind hospitality of T.U. Munich where this work was initiated.

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